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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

EMT1016 – ENGINEERING MATHEMATICS 1 (All Sections / Groups)

14 OCTOBER 2019
2:30 PM – 4:30 PM
(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **8 pages** (including cover page) with **4 Questions** only.
2. Attempt **all 4** questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Only NON-PROGRAMMABLE calculator is allowed.
5. Several tables are provided in the **Appendix** for your reference.

Question 1

(a) Evaluate the following limits, if they exist:

(i) $\lim_{x \rightarrow -\infty} \frac{\sqrt{16x^2 - 3}}{x + 9}$. [3 marks]

(ii) $\lim_{x \rightarrow 1} x^{\frac{1}{1-x}}$. [5 marks]

(b) A function is given below:

$$f(x) = \begin{cases} 5 - x, & 0 < x < 5, \\ (x - 5)^2, & x \geq 5. \end{cases}$$

Is $f(x)$ continuous at $x = 5$? Justify your answer.

[4 marks]

(c) Find the absolute maximum and minimum points of the given function:

$$f(x) = x - 2 \sin x, \text{ for } -\frac{\pi}{4} \leq x \leq \frac{\pi}{2}.$$

[6 marks]

(d) Use partial fraction decomposition to find $\int \frac{2x^2 - 9x - 9}{x^3 - 9x} dx$.

[7 marks]

Continued...

Question 2

- (a) The Taylor series of a function $f(x)$ is given by

$$\sum_{n=0}^{\infty} a_n (x-a)^n, \text{ for } x \in (a-R, a+R),$$

where $a_n = \frac{f^{(n)}(a)}{n!}$.

- (i) Find the first four terms of the Taylor series for $f(x) = e^{2x}$ about $x=0$.

[6 marks]

- (ii) Determine a general expression for the a_n in (i). Hence, find the radius of convergence of the series, given that

$$R = \lim_{n \rightarrow \infty} \frac{|a_n|}{|a_{n+1}|}.$$

[6 marks]

- (b) Let z and $\bar{z}$ denote a complex number and its complex conjugate, respectively.

- (i) If $z = 3 + i$, find the real values p and q that satisfy

$$2pz + q\bar{z} = 6z\bar{z}.$$

[5 marks]

- (ii) If $z = -2 + 2i$, express z in polar form. Hence, show that $z^8 = 2^{12}$.

[6 marks]

- (iii) If $z = i$, find $\text{Ln}(z)$.

[2 marks]

Continued...

Question 3

- (a) A periodic function $g(x)$ has the following Fourier series representation:

$$2 + \frac{40}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{n\pi x}{4}.$$

- (i) Determine the period of $g(x)$.

[2 marks]

- (ii) Is $g(x)$ an even function, odd function, or neither? Justify your answer.

[2 marks]

- (b) Suppose $g(x)$ in part (a) is multiplied with the squarewave $h(x)$ shown in Figure Q3 below:

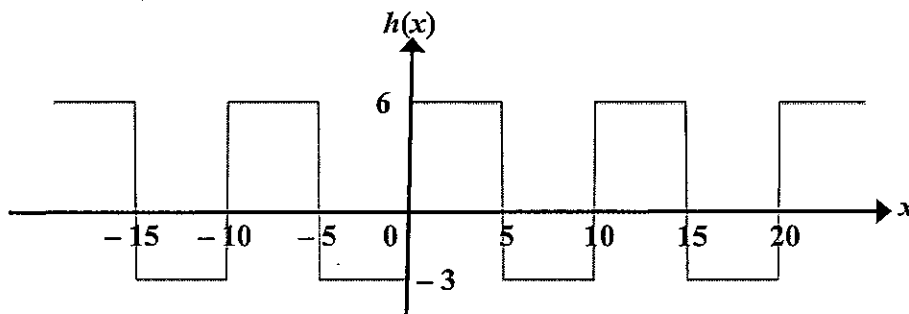


Figure Q3

- (i) What would be the period of $g(x) \times h(x)$?

[2 marks]

- (ii) If $h(x)$ was represented by a Fourier series, to what value would the series converge at $x = 5$?

[2 marks]

- (c) Let $f(x)$ be a periodic function of **period 6** which is defined by

$$f(x) = x^2 - 4, \text{ for } -3 \leq x < 3.$$

- (i) Sketch the graph of $f(x)$ from $x = -9$ to $x = 12$.

[4 marks]

- (ii) Develop the Fourier series expansion of $f(x)$.

[13 marks]

Continued...

Question 4

(Note: The tables in APPENDIX A, APPENDIX B and APPENDIX C may be useful for solving this question.)

- (a) The probability mass function of a random variable, X , is shown in Table Q4.1 below.

Table Q4.1

x	10	30	40	45	50
$P(X=x)$	0.1	a	b	0.3	0.2

Given that the mean, $E(X) = 38$.

- (i) Evaluate a and b . [4 marks]
 - (ii) Determine $P(10 < X \leq 45)$. [2 marks]
 - (iii) Find $E(X^2 - 5X + 2)$. [3 marks]
- (b) A certain equipment in a production factory malfunctions, on average, 4 times daily.
- (i) Find the probability that there are not more than two malfunctions on a Tuesday. [4 marks]
 - (ii) Find the probability that there will be exactly 25 malfunctions in a week. [2 marks]
- (c) Product ABC is sold online and it has two delivery partners, **Partner A** and **Partner B**. A customer is allowed to choose one of the delivery partners. The delivery time is normally distributed. Table Q4.2 stipulates the mean and standard deviation of delivery time (in days) for the partners.

Table Q4.2

	Partner A	Partner B
Mean	3 days	2.5 days
Standard deviation	1.5 days	2 days

- (i) Mr. Smith purchased Product ABC and he wants the product to arrive in less than two days. Which delivery partner would you recommend him? Please justify. [6 marks]
- (ii) Mdm. Husna purchased Product ABC on 1st May 2019, but she will not be home from 3rd to 4th May 2019 to receive any delivery. What is the probability that Partner B successfully delivered her order on the first attempt? [4 marks]

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APPENDIX A:
Special Discrete Probability Distributions

Binomial Distribution, $X \sim b(x, n, p)$	
P.m.f.	$P(X = x) = {}^n C_x p^x q^{n-x}$, for $x = 0, 1, \dots, n$ and where $q = 1 - p$.
Mean	$E[X] = np$
Variance	$Var(X) = npq$
Hypergeometric Distribution, $X \sim h(x, N, n, k)$	
P.m.f.	$P(X = x) = \frac{{}^k C_x \times {}^{N-k} C_{n-x}}{{}^N C_n}$, for $x = 0, 1, \dots, \min(n, k)$.
Mean	$E[X] = \frac{nk}{N}$
Variance	$Var(X) = \frac{nk}{N} \left(\frac{N-n}{N-1} \right) \left(1 - \frac{k}{N} \right)$
Poisson Distribution, $X \sim p(x, \lambda)$	
P.m.f.	$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$, for $x = 0, 1, \dots$
Mean	$E[X] = \lambda$
Variance	$Var(X) = \lambda$

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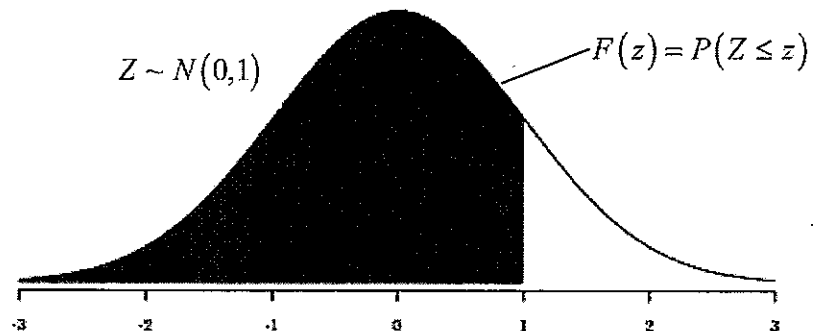
APPENDIX B:
Special Continuous Probability Distributions

Continuous Uniform Distribution, $X \sim U(a,b)$	
P.d.f.	$f_X(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \frac{a+b}{2}$
Variance	$Var(X) = \frac{(b-a)^2}{12}$
Exponential Distribution, $X \sim \exp(1/\beta)$	
P.d.f.	$f_X(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \beta$
Variance	$Var(X) = \beta^2$
Normal Distribution, $X \sim N(\mu, \sigma)$	
If X is any normal random variable where $X \sim N(\mu, \sigma)$, then the transformation $Z = \frac{X - \mu}{\sigma}$ yields a <i>standard</i> normal variable where $Z \sim N(0,1)$.	

Continued...

APPENDIX C:

Cumulative Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

End of paper.